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Dynamic scaling theory for a tethered membrane in solution

Sheh-Yi Sheu[1] and Dah-Yen Yang[2]
[1]*Department of Life Science, National Yang-Ming University, Taipei, Taiwan*
[2]*Institute of Atomic and Molecular Science, Academia Sinica, Taipei, Taiwan*

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We present the dynamic scaling behavior for the specific viscosity and diffusion coefficient of a single membrane and membranes with nonzero concentration in solution. Starting from the membrane free energies, we derive their Langevin equations. The corresponding Kirkwood diffusion equation, describing the time evolution in configuration space, contains two kinds of time scales that are separated by the external dimension $4/(2-D)$ where D is the dimension of the internal space. These time scale separation behaviors depend strongly on the hydrodynamic screening effect. For a single membrane solution, we resolve the dynamic scaling exponents for the diffusion coefficient and intrinsic viscosity by the dimension reduction method. For a concentrated membrane solution, the effective excluded volume strength and draining parameter are introduced. The effective medium argument is applied to obtain a concentration dependent power law form for the specific viscosity and diffusion coefficient, whose results contribute to a fundamental understanding of membrane in solution and of hydrodynamic screening and excluded volume effects in many different solvents.

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I. INTRODUCTION

Scaling behavior is a widely accepted concept, and exists in many physical systems such as magnetic, polymer, membrane, and disordered systems. The static scaling theory of polymer solutions was initiated in Refs. [1–7]. These theories provide power laws for various quantities such as the structure factor and the second virial coefficient. A simple and systematic derivation was given by Kosmas and Freed [8]. In order to generalize the static scaling result, the authors of Refs. [9–14] introduced the dynamic scaling theory for polymer solutions. Moreover, Adler and Freed (AF) [15] gave a systematic derivation of the dynamic scaling theory for the diffusion coefficient and specific viscosity by applying the Kirkwood diffusion equation to polymers. Their single polymer theory is based upon a real space Oseen tensor, and is valid for an external space dimension $d<4$. Similarly, Marqusee and Deutch [16] (MD) extended the AF method to $d>4$.

A tethered membrane is a general extension of a polymer, and has attracted a great deal of attention within the last two decades. Its fundamental static scaling behavior has been studied by scaling theory [17], and static renormalization group theory [18–20]. Moreover, dynamic behaviors of the membrane, such as the dynamic radius of gyration exponent and the single monomer diffusion exponent were studied in Refs. [21–26].

In the present paper, we generalize our previous work [17] to study the dynamic scaling behavior of a tethered membrane dissolved in solution. It is well known that the dynamic properties of a polymer in dilute solution are usually treated as Brownian motion. Based on the generalized membrane free energy suggested by Yang and Sheu [17], we derive a Langevin equation for membrane dynamics. An equivalent Fokker-Planck equation can then be elaborated. However, the membrane Kirkwood diffusion equation contains two time scales. The choice of these time scales depends on the positivity of the power of the bead number in the draining parameter. We perform the dynamic scaling behavior of the diffusion coefficient and specific viscosity for both a single membrane and concentrated membranes dissolved in solution. In this context, the difference between a polymer solution and a membrane solution is shown in the dynamic critical exponent of a single entity (see Table I) and in the Huggins coefficient of a dilute solution (see Table II).

The paper is organized as follows: In Sec. II, we first describe the Langevin equation and its corresponding Kirkwood diffusion equation for a membrane dissolved in solution. The scaling behaviors of the diffusion coefficient, intrinsic viscosity, and dynamic structure factor for a single membrane are presented in Sec. III. We subsequently apply the k-space Oseen tensor in Sec. IV, which was suggested by MD to elucidate the dynamic scaling law of the diffusion coefficient and specific viscosity for a membrane solution in a finite concentration. Thus, in θ solvents and good solvents, the hydrodynamic interaction and the excluded volume interaction have to be renormalized. Based on the effective medium argument, the concentration dependent part of the interactions exhibits a power law behavior. Our finite concentration results cover the limiting law behaviors for both the diffusion constant and specific viscosity.

TABLE I. Dynamical critical exponent for one polymer and one membrane. ω_d and $\overline{\omega}_d$ are defined in Sec. III.

Dynamical critical exponent	Polymer[a]	Membrane[b]
ω_d	ν_d	$(d-2)\nu$
$\overline{\omega}_d$	ν_d	$\frac{d}{3}\nu$

[a] $\nu_d=3/(2+d)$ in Ref. [15].
[b] ν in Eq. (13).

TABLE II. Huggins coefficient of dilute polymer and membrane solutions (in a good solvent). The related notations are explained in Sec. IV.

	Huggins coefficient	
Dimension	Polymer	Membrane
	for specific viscosity	
$d<\frac{4}{2\text{-}D}$	$\alpha_1-3(2\nu_d-1)\alpha_2$	$\alpha_1-\frac{d\nu-(2-D)d/2}{nD-(n-1)(2-D)d/2}\alpha_2$
$d>\frac{4}{2-D}$	$\alpha_1'-2(3\nu_d-2)\alpha_2'$	$\alpha_1'-\frac{d\nu-2}{nD-(n-1)(2-D)d/2}\alpha_2'$
	for diffusion coefficient	
$d<\frac{4}{2-D}$	$\overline{\alpha_1}-(2\nu_d-1)\overline{\alpha_2}$	$\overline{\alpha_1}+\frac{(2-d)[\nu-(2-D)/2]}{nD-(n-1)(2-D)d/2}\overline{\alpha_1}$
$d>\frac{4}{2-D}$	$\overline{\alpha_1'}-2(\nu_d-1)\overline{\alpha_2'}$	$\overline{\alpha_1'}+\frac{D-(d-2)\nu}{nD-(n-1)(2-D)d/2}\overline{\alpha_2'}$

II. KIRKWOOD DIFFUSION EQUATION OF A TETHERED MEMBRANE

We first consider a continuous model of a flexible polymerized membrane with a D manifold flat internal space R^D, embedded by $\vec{R}(x)$ in a d-dimensional external space R^d, where $x \in R^D$ (see Fig. 1). This model is a general extension of Edward's model of a continuous polymer chain. The associated action or free energy for a single membrane configuration is given by

$$F(\{\vec{R}\},L,l,v_n)/k_BT = \frac{1}{2}\kappa\int_0^{L^D} d^Dx \sum_{a=1}^{D}\left(\frac{\partial\vec{R}}{\partial x_a}\right)^2 + \frac{1}{n!}v_n\int\prod_{i=1}^{n} d^Dx_i \prod_{j=1}^{n-1} \delta^d(\vec{R}(x_j)-\vec{R}(x_n)), \qquad (1)$$

where $\vec{R}(x)$ denotes the position vector on the membrane from an arbitrary origin to a point x along the membrane. Here L corresponds to the extended length and $l=\kappa^{-1/D}$ is the Kuhn length in each dimension, v_n is the excluded volume strength, and k_BT is the temperature in units of energy. In Eq. (1), the first term is the Gaussian elastic energy and the interaction term denotes the n-body excluded volume effect for $v_n>0$. In our previous paper [6], we showed that the variable transformations $\vec{R}'=b^{(2-D)/2}l^{-1}\vec{R}$ and $x'=bl^{-1}x$, when applied to F, result in the following homogeneity relation of the free energy $F=F(\{\vec{R}'\}, L'=Nb, l'=1, v_n'=v_nl^{(n-2)D-(n-1)d}b^{(n-1)(2-D)d/2-nD})$, where N is the bead number in each dimension. Furthermore, the corresponding time dependent properties of the membrane solution can be described in terms of the conditional probability $P(\{\vec{R}\},t;\{\vec{R}_0\},0)$ such that a membrane has the configuration $\{\vec{R}\}$ at time t with the initial configuration $\{\vec{R}_0\}$. Hence the evolution of P is governed by the Kirkwood diffusion equation which is derived from a stochastic process.

To begin with a description of membrane dynamics in solution, we consider the coupled Langevin equations for the membrane and solvent. First we employ the membrane conformation $\vec{r}(x,t)=\sqrt{\kappa}\vec{R}(x,t)$. At any beads of the membrane, the friction force is balanced by the membrane-membrane interaction and random forces. We can then immediately write down the Langevin equation

$$\frac{\partial}{\partial t}\vec{r}(x,t)=\vec{u}(\vec{r}(x,t),t)-\frac{k_BT\kappa}{\zeta_0}\frac{\delta\overline{F}}{\delta\vec{r}(x,t)}+\vec{\theta}(x,t) \qquad (2)$$

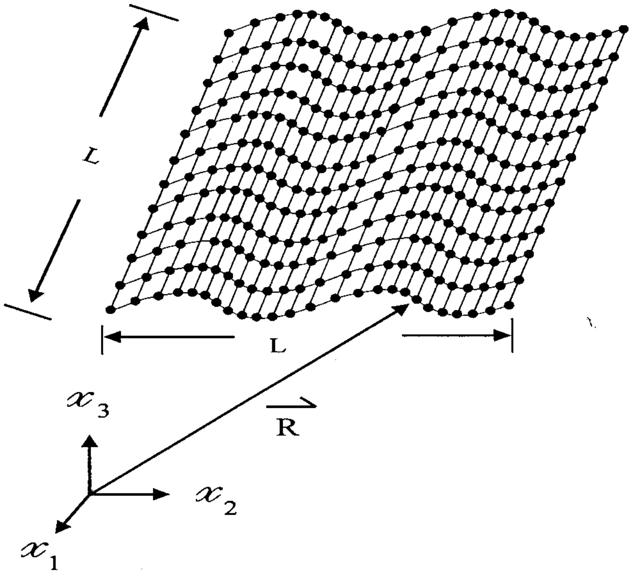

FIG. 1. Self-avoiding tethered membrane with length L.

where ζ_0 is the bare friction coefficient for a unit membrane area, and $\vec{u}$ is the effective fluctuating solvent velocity field at position $\vec{r}$ and time t; $\overline{F}$ is equivalent to F/k_BT. We refer to $\vec{\theta}(x,t)$ term as a random force whose components satisfy the Gaussian white noise spectrum with zero mean and the variance relation

$$\langle \theta_\alpha(x_1,t)\,\theta_\beta(x_2,t)\rangle = \frac{2k_BT}{\zeta_0}\,\delta^D(x_1-x_2)\,\delta(t-t')\,\delta_{\alpha\beta},$$

$$\alpha,\beta=1,\dots,d. \tag{3}$$

For a polymer dissolved in solution, the above variance of random force is then reduced to a scalar form $\langle \theta(x_1,t)\,\theta(x_2,t)\rangle = (2k_BT/\zeta_0)d\,\delta(x_1-x_2)\,\delta(t-t')$, where d indicates the dimension in Eq. (3).

Now we consider the solvent velocity field. It is well known that the fluctuating solvent velocity field obeys the Navier-Stokes equation, which contains the friction forces due to the membrane and the random thermal velocity fluctuation in the fluid,

$$\rho\frac{\partial}{\partial t}\vec{u}(\vec{y},t) = \eta_0\vec{\nabla}^2\vec{u}(\vec{y},t) - \vec{\nabla}p - \frac{k_BT\kappa}{l^D} \times \int_0^{L^D} d^Dx\,\frac{\delta\overline{F}}{\delta\vec{r}(x,t)}\,\delta^d[\vec{y}-\vec{R}(x,t)] + \vec{f}(\vec{y},t), \tag{4}$$

where ρ is the solvent density, η_0 is the solvent viscosity and the position vector $\vec{y}\in R^d$. Here p is the effective hydrostatic pressure in the fluid. $\vec{f}$ is the hydrodynamic fluctuation random force density with zero mean, and its variance satisfies

$$\langle f_\alpha(\vec{y},t)f_\beta(\vec{y}',t')\rangle = -2k_BT\kappa\,\eta_0\vec{\nabla}^2\delta^d(\vec{y}-\vec{y}')\,\delta(t-t')\,\delta_{\alpha\beta}. \tag{5}$$

Furthermore, the solvent velocity field $\vec{u}(\vec{y},t)$ obeys the incompressibility condition: $\nabla\cdot\vec{u}=0$.

In order to discuss the membrane dynamics, it is useful for us to transform the preceding stochastic Langevin equation into an equivalent probability description, such as the corresponding Kirkwood diffusion equation in configuration space. The transformation of the Langevin equation into the Kirkwood diffusion equation is carried out by a k-space method, and the Fourier transform of the Naiver-Stokes equation is defined by $\vec{u}_{\vec{k}}(t)=\int d^dy\,e^{i\vec{k}\cdot\vec{y}}\vec{u}(\vec{y},t)$. We eliminate the pressure term and obtain the transversal components of Eq. (4), that implies the incompressibility $\vec{k}\cdot\vec{u}_k=0$, such as

$$\rho\frac{\partial}{\partial t}\vec{u}_{\vec{k}}(t) = \left[\mathbb{1}-\frac{\vec{k}\vec{k}}{k^2}\right]\left[-\eta_0\kappa k^2\vec{u}_{\vec{k}} - \frac{k_BT\kappa}{l^D} \times \int d^Dx\,\frac{\delta\overline{F}}{\delta\vec{r}(x,t)}\,e^{i\vec{k}\cdot\vec{R}(x,t)}\right] + \vec{f}^{\,\perp}_{\vec{k}}(t). \tag{6}$$

Here $\mathbb{1}$ is a $(d\times d)$-dimensional tensor matrix, and the superscript $\perp$ denotes the vector components orthogonal to the k direction.

According to the method established in some detailed by Ma and Mazenko [27], the above time-dependent Landau-Ginzberg equation can be transformed into the Kirkwood diffusion equation by averaging out the noises, i.e., $\vec{\theta}$ and $\vec{f}$. Consequently, the Kirkwood diffusion [28] equation turns out to be

$$\frac{\partial}{\partial t}P(\{\vec{R}\},t) = \left\{\int_0^{L^D\times L^D} d^Dx\,d^Dx'\,\frac{\delta}{\delta\vec{R}(x,t)}\left[\mathbb{1}\delta^D(x-x')\,\frac{\kappa_BT}{\zeta_0} + \rho_0^2\frac{k_BT}{\eta_0}T(\vec{R}(x)-\vec{R}(x'))\right] \times\left[\frac{\delta}{\delta\vec{R}(x',t)}+\frac{\delta\overline{F}}{\delta\vec{R}(x',t)}\right]\right\}P(\{\vec{R}\},t) \tag{7}$$

where $T(\vec{R}(x)-\vec{R}(x)'))=(2\pi)^{-d}\int d^dk\,\exp[i\vec{k}\cdot(\vec{R}(x)-\vec{R}(x'))]k^{-2}$ is the real space Oseen tensor that is a fundamental quantity in all dimensions, and ρ_0 is the mode coupling constant.

Equation (7) is the starting point of our dynamical scaling analysis. However, the time scale in Eq. (7) is not unique. There are two time scales such as

$$t' = t\,\frac{k_BT}{\eta_0}\left(\frac{b^{(2-D)/2}}{l}\right)^d$$

from the $\rho_0^2(k_BT/\eta_0)T(\vec{R}(x)-\vec{R}(x'))$ term and $t''=t(k_BT/\zeta_0)(b^2/l^{2+D})$ from the $\delta^D(x-x')(k_BT/\zeta_0)$ term. This makes the study of membrane dynamics quite complicated. The choice of these time scales depends on the membrane concentration. In addition, the criteria of determining which time scale is suitable is based on the following arguments. When the concentration c is equal to zero, we choose t' for $d<4/(2-D)$ and t'' for $d>4/(2-D)$. On the contrary, when $c\neq 0$, we use t'' for $d<4/(2-D)$ and t' for $d>4/(2-D)$. The reason for this is due to the asymptotic behavior of the bead number N in the draining parameter, i.e., the draining parameter has to converge when $N\to\infty$.

III. DYNAMIC SCALING FOR A SINGLE MEMBRANE

In this section, we begin by considering some basic facts on the dynamic scaling behavior of the diffusion coefficient, intrinsic viscosity, and dynamic structure factor for a single membrane dissolved in Θ and good solvents.

A. Diffusion coefficient

According to the Kirkwood diffusion equation, we can see that the membrane dynamical behavior is governed by the set of membrane and solvent dependent parameters $\{k_BT/\zeta_0,\ k_BT/\eta_0,\ L,\ \kappa,\ \text{and}\ v_n\}$. In this system, the time scale of the solvated membrane system follows the transformation t' or t''. Linear response theory predicts that the single membrane diffusion coefficient D_d may be written as

$$D_d=\frac{1}{d}\int_0^{\infty} dt\langle V(t)V(0)\rangle, \tag{8}$$

where $V(t)$ is the velocity of the center of mass of the membrane, and is equal to $(1/L^D)\int_0^{L^D} d^D x(\partial/\partial t)\vec{R}(x,t)$. The term in angular brackets in Eq. (8) indicates the ensemble average, and is expressed in terms of the probability distribution that is obtained by solving the Kirkwood diffusion equation.

To write down the scaling function, we substitute Eq. (7) into Eq. (8), which results in the dynamic scaling relationships

$$\begin{aligned}
D_d=&\frac{k_BT}{\eta_0 l^{d-2}}N^{-2D}b^{(d/2-1)(2-D)-2D}\\
&\times f_D\Bigg(\frac{\eta_0}{\zeta_0}\frac{b^{D+(2-D)(2-d)/2}}{l^{D+2-d}},Nb,v_n\\
&\times l^{(n-2)D-(n-1)d}b^{(n-1)(2-D)d/2-nD}\Bigg),\\
&\quad\text{for } d<\frac{4}{2-D}\\
=&\frac{k_BT}{\zeta_0}\frac{1}{(Nb)^2}\frac{b^D}{l^D}\\
&\times f_D\Bigg(\frac{\zeta_0}{\eta_0}l^{2+D-d}b^{-2+d(2-D)/2},v_n l^{(n-2)D-(n-1)d}\\
&\times b^{(n-1)(2-D)d/2-nD}\Bigg)\quad\text{for } d>\frac{4}{2-D}.
\end{aligned} \tag{9}$$

The above equation is true for any $b>0$. Hence making the selection $Nb=1$ yields the diffusion coefficient

$$\begin{aligned}
D_d=&\frac{k_BT}{\eta_0 l^{d-2}}N^{(d/2-1)(D-2)}f_D(Y^{-1},Z)\\
&\qquad\text{for } d>\frac{4}{2-D}\\
=&\frac{k_BT}{\zeta_0}\frac{N^{-D}}{l^D}f_D(Y,Z)\quad\text{for } d>\frac{4}{2-D}.
\end{aligned} \tag{10}$$

Here, for our convenience, we refer to Z as $v_n l^{-d}N^{nD-(n-1)(2-D)d/2}$ and to Y as $(\zeta_0/\eta_0)l^{2+D-d}N^{2-(2-D)d/2}$.

The scaling functions in Eq. (10) contain the hydrodynamic interaction and excluded volume effect. This deserves more exploration. Recall that, in Eq. (4.1) of Ref. [5] when the internal dimension $d>4$ and in the limit of large bead number in each dimension $N\rightarrow\infty$, both the excluded volume effect and the hydrodynamic interaction are irrelevant for the polymer case. In fact, in our membrane system, the situation is more complicated. The hydrodynamic interaction (the Y term) approaches zero when $N\rightarrow\infty$. On the other hand, the excluded volume strength (the Z term) also vanishes while $2nD<(n-1)(2-D)d$. Hence both the excluded volume effect and the hydrodynamic interaction are irrelevant when the dimensional lower bound satisfies $2/(4/d+1)>D$ for $n=2$. This is denoted as a freely draining Rouse type behavior. Moreover, the monomer diffusion coefficient is irrelevant for large N when $d<4/(2-D)$. If $D>[2(d-2)]/d$ and $v_n=0$ (Θ solvents), the diffusion coefficient D_d of a single membrane is reduced to the well known nonfree draining Rouse-Zimm result

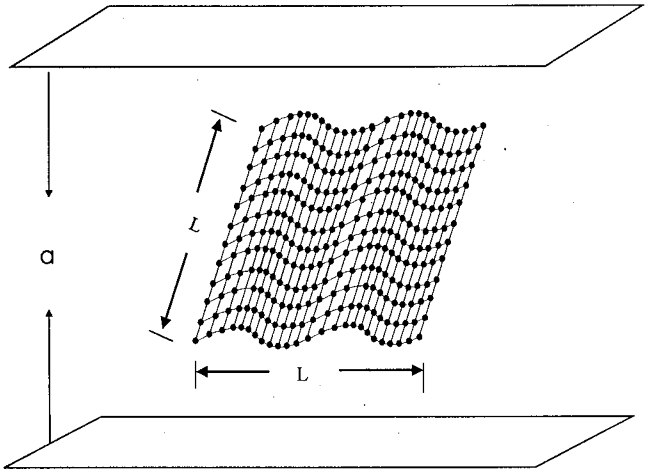

FIG. 2. A membrane confined between two infinite planes distance a apart.

$$D_d=\frac{k_BT}{\eta_0 l^{d-2}}N^{-(d/2-1)(2-D)}f_D(0). \tag{11}$$

In addition, in good solvents, the excluded volume effect sets in, and $Z\gg 1$. If we take $N\rightarrow\infty$, the membrane diffusion coefficient becomes $D_d=(k_BT/\eta_0 l^{d-2})N^{(d/2-1)(D-2)}f_D(Z)$. However, in good solvents, the diffusion coefficient displays a power-law dependence $D_d\sim N^{-\omega_d}$. We require that f_D is determined by its argument through a power law form. Assuming that the scaling function $f_D(Z)$ scales as Z^w, we recognize that the scaling relation of the diffusion coefficient obeys a relationship such as

$$\begin{aligned}
D_d=&c_D\frac{k_BT}{\eta_0 l^{d-2}}\\
&\times N^{(d/2-1)(D-2)}Z^{-[\omega_d+(d/2-1)(D-2)]/[nD-(n-1)(2-D)d/2]},
\end{aligned} \tag{12}$$

where ω_d is a new dynamic scaling exponent.

To solve the dynamic scaling exponent, we consider a membrane confined between two infinite planes with a distance a apart (see Fig. 2). The scaling relation of the confined membrane must be reduced to the unconfined result as $a\rightarrow\infty$, assuming that there exists only one characteristic length for the membrane diffusing a distance R. This characteristic length enables us to obtain the scaling relation for the confined membrane in d dimensions, i.e.,

$$\begin{aligned} D_d = & \frac{k_B T}{\eta_0} N^{-\omega_d} l^{2-d} \\ & \times (v_n l^{(n-2)D-(n-1)d})^{[(d/2-1)(2-D)-\omega_d]/[nD-(n-1)(2-D)d/2]} \\ & \times g_D(a/R) \end{aligned}$$

for $d < 4/(2-D)$, where $g_D[(a\rightarrow\infty)/R=\infty]=\text{const}$.

At this stage, we can truncate the diffusion coefficient to a lower dimension. In the limit $a/R \rightarrow 0$, the scaling behavior of the confined-membrane is the same as the scaling behavior of the unconfined membrane in $(d-1)$ dimensions. Assuming that we have a power law form for the scaling function $g_D(q)$ as q^w. The same results are obtained by assuming that R is the static radius of gyration R_G, or a dynamic length R_D defined by the Stokes-Einstein relation $D_d = \text{const}\times(k_B T/\eta_0)(1/R_D^{d-2})$. This consideration gives us the static radius of gyration $R_G^2 \sim l^D L^{2-D} Z^{w_1}$, where

$$w_1 = \frac{2\nu-(2-D)}{nD-(n-1)(2-D)d/2}.$$

So we obtain we obtain the characteristic length

$$R_G \sim N^{\nu} v_n^{[2\nu-(2-D)]/[2nD-(n-1)(2-D)d]} \tag{13}$$

where $\nu=(2-D)/2$ plus a correction term [6].

Next we can investigate ω_d. We first proceed with the static definition of R_G and find that the scaling law for the confined membrane in d dimensions in the limit $a/R_G \rightarrow 0$ becomes

$$\begin{aligned} D_d \sim & N^{-\omega_d} v_n^{[(d/2-1)(2-D)-\omega_d]/[nD-(n-1)(2-D)d/2]} \\ & \times N^{-w_2 \nu} v_n^{-w_2[2\nu-(2-D)]/[2nD-(n-1)(2-D)d]}. \end{aligned}$$

Equating the powers of N and v_n in d-dimensional D_d with the related powers in $(d-1)$-dimensional D_{d-1}, we then reduce the diffusion coefficient into

$$D_{d-1} = \text{const}\times(k_B T/\eta_0) l^{2-D} N^{-\omega_{d-1}} (v_n l^{(n-2)D-(n-1)(d-1)})^{\{[(d-1)/2-1](2-D)-\omega_{d-1}\}/[nD-(n-1)(2-D)(d-1)/2]},$$

where the relationships

$$\omega_{d-1} = \omega_d + w_2 \nu \tag{14}$$

and

$$\begin{aligned} & \frac{(d/2-3/2)(2-D)-\omega_{d-1}}{nD-(n-1)(2-D)(d-1)/2} \\ & \quad = \frac{(d/2-1)(2-D)-\omega_d}{nD-(n-1)(2-D)d/2} - w_2 \frac{2\nu-(2-D)}{nD-(n-1)(2-D)d} \end{aligned} \tag{15}$$

are satisfied. By eliminating w_2 from the previous equations, it may be verified that the first-order difference equation for ω_d obeys the relation

$$\begin{aligned} & \frac{(d-3)(2-D)-2\omega_{d-1}}{2nD-(n-1)(2-D)(d-1)} \\ & \quad = \frac{(d-2)(2-D)-2\omega_d}{2nD-(n-1)(2-D)d} \\ & \quad\quad - \frac{(\omega_{d-1}-\omega_d)(2\nu-(2-D))}{\nu(nD-(n-1)(2-D)d)}. \end{aligned} \tag{16}$$

The above equation shows a recurrence relationship between ω_d and ω_{d-1}. Continuing this iteration process, the recurrence relation is terminated at ω_1. Recall that the boundary condition ω_1 should be able to reproduce the polymer result when $n=2$ and $D=1$. Typically, in the polymer case and in good solutions, the hydrodynamic interaction range is confined in a very short range, say a few monomers. In other words, the range of the hydrodynamic interaction is of the order of the polymer tube diameter. The translational diffusion coefficient for a Rouse chain is proportional to the inverse of N, i.e., $\omega_1 = 1$ for $n=2$ and $D=1$. Therefore, putting the preceding boundary condition into Eq. (16) yields the more general boundary condition for the membrane case

$$\omega_1 = nD - n(2-D)/2 \tag{17}$$

for arbitrary n and D. Finally, we substitute Eq. (17) into Eq. (16) and obtain the general dynamic critical exponent

$$\omega_d = (d-2)\nu. \tag{18}$$

The above equation covers the polymer result of AF. In a two-dimensional external space, $\omega_d = 0$ and the diffusion coefficient D_2 is independent of N.

B. Intrinsic viscosity

The other interesting physical quantity for a membrane solution is the intrinsic viscosity, which for a single membrane is calculated by linear response theory as

$$[\eta] = \frac{N_A}{M k_B T \eta_0} \int_0^{\infty} dt \langle J(t) J(0) \rangle, \tag{19}$$

where N_A is Avogadro's number and M is the molecular weight of the membrane; the related momentum flux tensor is defined by $J(t) = -k_B T \int_0^{L^D} d^D x \vec{r}_x(x,t)[\delta/\delta\vec{r}_y(x,t)]F$. In general, the scaling relation for the intrinsic viscosity may be obtained by substituting Eq. (7) into Eq. (19) and leads us to the scaling functions

$$\begin{aligned}[\eta] &= \frac{N_A}{M} l^d b^{-d(2-D)/2} f_\eta\Bigg(Nb, \frac{\eta_0}{\zeta_0}\frac{b^{D+(2-D)(2-d)/2}}{l^{D+2-d}}, \\ &\quad v_n l^{(n-2)D-(n-1)d} b^{(n-1)(2-D)d/2-nD}\Bigg) \\ &\qquad \text{for } d<\frac{4}{2-D} \\ &= \frac{N_A\zeta_0}{M\eta_0}\frac{l^{2+D}}{b^2} \\ &\quad \times f_\eta\Bigg(Nb, \frac{\zeta_0}{\eta_0} l^{2+D-d} b^{-2+d(2-D)/2}, \\ &\quad v_n l^{(n-2)D-(n-1)d} b^{(n-1)(2-D)d/2-nD}\Bigg), \\ &\qquad \text{for } d>\frac{4}{2-D}. \end{aligned} \tag{20}$$

By setting $Nb=1$, Eq. (20) is simplified to

$$\begin{aligned}[\eta] &= \frac{N_A}{M} l^d N^{d(2-D)/2} f_\eta(Y^{-1},Z) \quad \text{for } d<\frac{4}{2-D} \\ &= \frac{N_A\zeta_0}{M\eta_0} l^{2+D} N^2 f_\eta(Y,Z) \quad \text{for } d>\frac{4}{2-D}. \end{aligned} \tag{21}$$

In higher dimension, due to the same argument suggested for the diffusion coefficient, both the excluded volume effect and the hydrodynamic interaction are irrelevant. Hence we obtain the freely draining Rouse behavior for Θ solvents as

$$[\eta] = \frac{N_A}{M}\frac{\zeta_0}{\eta_0} l^{2+D} N^2 f_\eta(0). \tag{22}$$

Conversely, in lower dimension and in Θ solvents, Eq. (21) reduces to

$$[\eta] = \frac{N_A}{M} l^d N^{d(2-D)/2} f_\eta(0), \tag{23}$$

when $N\to\infty$. This is the general Rouse-Zimm result.

For good solvents, $[\eta]$ displays a different power law form from N. We require the intrinsic viscosity scaling relationship $[\eta]\sim N^{3\overline{\omega}_d}$, i.e.,

$$\begin{aligned}[\eta] &= \frac{N_A}{M} l^3 N^{3\overline{\omega}_d} \\ &\quad \times (v_n l^{(n-2)D-(n-1)d})^{[3\overline{\omega}_d-d(2-D)/2]/[nD-(n-1)(2-D)d/2]} g_\eta. \end{aligned}$$

Here ω_d is a new dynamic scaling exponent of the intrinsic viscosity.

We can re-express the g_η part in terms of a power law form q^w. Then, the intrinsic viscosity becomes

$$\begin{aligned}[\eta] &= \text{const}\times(N_A/M) l^3 N^{3\overline{\omega}_d-w_3\nu}\times l^{[(n-2)D-(n-1)d][3\overline{\omega}_d-d(2-D)/2]/[nD-(n-1)(2-D)d/2]} \\ &\quad \times v_n^{[3\overline{\omega}_d-d(2-D)/2]/[nD-(n-1)(2-D)d/2]-w_3[2\nu-(2-D)]/[2nD-(n-1)(2-D)d]}. \end{aligned}$$

This enables us to determine the recurrence relationships of the powers

$$3\overline{\omega}_{d-1} = 3\overline{\omega}_d - w_3\nu \tag{24}$$

and

$$\begin{aligned}\frac{3\overline{\omega}_{d-1}-(d-1)(2-D)/2}{nD-(n-1)(2-D)d-1)/2} &= \frac{3\overline{\omega}_d-d(2-D)/2}{nD-(n-1)(2-D)d/2} \\ &\quad - w_3\frac{2\nu-(2-D)}{2nD-(n-1)(2-D)d}. \end{aligned} \tag{25}$$

Following the same argument in Sec. III A, we obtain the remarkable boundary condition for the membrane case as

$$3\overline{\omega}_1 = nD/3-(n-4)(2-D)/6, \tag{26}$$

with the requirement of Rouse-like behavior when $[\eta]$ is reduced to the polymer case in good solvents. We therefore have the general dynamic exponent

$$3\overline{\omega}_d = d\nu. \tag{27}$$

Perhaps the most striking feature is that when the external dimension is larger than one, both the dynamic exponents in Eqs. (18) and (27) are independent of n and the internal dimension. Note, in particular, that $\overline{\omega}_d$ (or ω_d) is the same for polymers and membranes in dimension $d=3$.

C. Dynamic structure factor

The experimentally measured coherent neutron and light scattering signal intensities, which carry information pertinent to density-density correlations in membrane solutions, are proportional to the spectrum structure factor

$$S(\vec{k},\omega)=2\ \mathrm{Re}\lim_{\varepsilon\to 0}\int_0^{\infty}dt\langle\rho(\vec{k},t)\rho(-\vec{k},0)\rangle e^{-(i\omega+\varepsilon)t}, \tag{28}$$

where the density distribution is defined by the Fourier transform of the configuration vector $\rho(\vec{k},t)=(1/L^D)\int_0^{L^D}d^Dx\exp(i\vec{k}\cdot\vec{R}(x,t))$. Using now the correspondence between the structure factor and the diffusion coefficient, the half-width of the Lorentzian distribution of the structure factor

$$S(\vec{k},\omega)=2\ \mathrm{Re}[i\omega+k^2D_d(\vec{k},\omega)]^{-1}\langle|\rho(\vec{k},0)|^2\rangle \tag{29}$$

for a single membrane density gives the generalized diffusion coefficient. Here $\vec{k}=|k|$.

Now we can evaluate the scaling behavior of $D_d(\vec{k},\omega)$. We first set $N=L/l$ and the Fourier transform of the density distribution is expressed as

$$\rho(\vec{k},t)=\frac{1}{(Nb)^D}\int_0^{(Nb)^D}d^Dx'\ \exp[i\vec{k}lb^{(D-2)/2}\cdot\vec{R}'(x',t')].$$

Using the time scale relationships

$$\omega' t'=\omega' t\frac{k_BTb^{(2-D)/2}}{\eta_0 l^d}=\omega t$$

and

$$\omega''=\omega\frac{\zeta_0}{k_BT}l^{2+D}b^{-2},$$

we arrive at the following scaling relations of the dynamic structure factor as

$$\begin{aligned}\frac{S(\vec{k},\omega)}{\langle|\rho(\vec{k},0)|^2\rangle}&=2\ \mathrm{Re}\Bigg[i\omega+k^2\frac{k_BT}{\eta_0 l^{d-2}}b^{(1-d/2)(D-2)}f_D\Bigg(klb^{(D-2)/2},\omega\frac{\eta_0 l^d}{k_BT}b^{(D-2)d/2},Nb,\frac{\eta_0}{\zeta_0}\frac{b^{D+(2-D)(2-d)/2}}{l^{D+2-d}},\\&\qquad v_n l^{(n-2)D-(n-1)d}b^{(n-1)(2-D)d/2-nD}\Bigg)\Bigg]^{-1}\quad\text{for } d<\frac{4}{2-D}\\&=2\ \mathrm{Re}\Bigg[i\omega+k^2l^{-D}b^Df_D\Bigg(klb^{(D-2)/2},\omega\frac{\zeta_0}{k_BT}l^{2+D}b^{-2},Nb,\frac{\zeta_0}{\eta_0}l^{2+D-d}b^{d(2-D)/2-2},\\&\qquad v_n l^{(n-2)D-(n-1)d}b^{(n-1)(2-D)d/2-nD}\Bigg)\Bigg]^{-1}\quad\text{for }\ d>\frac{4}{2-D}.\end{aligned}$$

By making the selection $b=N^{-1}$, the above equation permits the construction of a general scaling form for D_d:

$$\begin{aligned}D_d(\vec{k},\omega)&=\frac{k_BT}{\zeta_0}l^{-D}N^{-D}f_D\left(klN^{(2-D)/2},\omega\frac{\zeta_0}{k_BT}l^{2+D}N^2,Z\right)\quad\text{for }\ d>\frac{4}{2-D}\\&=\frac{k_BT}{\eta_0 l^{d-2}}N^{(d/2-1)(D-2)}f_D\left(klN^{(2-D)/2},\omega\frac{\eta_0}{k_BT}l^dN^{d(2-D)/2},Z\right)\quad\text{for }\ d<\frac{4}{2-D}.\end{aligned}\tag{31}$$

Here the factor Y has been ignored as $N\to\infty$. For long wavelength, low frequency, and Θ solvents, we can see that the result of the Rouse-Zimm behavior in Eq. (10) is reobtained. In good solvents, $Z\gg 1$, we know that the membrane dimension is characterized by the crumpled dimension R_G in Eq. (13) instead of by the unperturbed dimension $N^{(2-D)/2}l$. Hence we obtain the generalized diffusion coefficient as

$$D_d(\vec{k},\omega)=\frac{k_BT}{\eta_0 R_G^{d-2}}f_D\left(kR_G,\omega\frac{\eta_0}{k_BT}R_G^d\right). \tag{32}$$

Conversely, in the long wavelength, low frequency regime, Eq. (12) is recovered. For a large momentum transfer $klN^{(2-D)/2}\gg 1$, and only the local property can be detected. In this respect, the generalized diffusion coefficient is N independent. This results in the well known simplified dynamic scaling relationship for the generalized diffusion coefficient as

$$D_d(\vec{k},\omega)=\left\{\begin{array}{ll}\dfrac{k_BT}{\eta_0}kf_{D,1}\!\left(\omega\dfrac{\eta_0}{k_BT}k^{-d}\right), & v_n=0\\ \dfrac{k_BT}{\eta_0}kf_{D,2}\!\left(\omega\dfrac{\eta_0}{k_BT}k^{-d}\right), & Z\gg 1\\ \dfrac{k_BT}{\eta_0}k,\omega\dfrac{\eta_0}{k_BT}k^{-d}\ll 1 & \end{array}\right\}\quad \text{for}\quad d<\frac{4}{2-D}$$

$$=\left\{\begin{array}{ll}\dfrac{k_BT}{\zeta_0}k^{(2-D)/2}l^{(2-3D)/2}f_{D,1}\!\left(\omega\dfrac{\zeta_0}{k_BT}k^{D-2}l^{2D}\right), & v_n=0\\ \dfrac{k_BT}{\zeta_0}k^{(2-D)/2}l^{(2-3D)/2}f_{D,2}\!\left(\omega\dfrac{\zeta_0}{k_BT}k^{D-2}l^{2D}\right), & Z\gg 1\\ \dfrac{k_BT}{\zeta_0}k^{(2-D)/2}l^{(2-3D)/2},\omega\dfrac{\zeta_0}{k_BT}k^{D-2}l^{2D}\ll 1 & \end{array}\right\}\quad \text{for}\quad d>\frac{4}{2-D}. \tag{33}$$

When we check the above equation, we find that, in a lower dimension, the dynamic diffusion coefficient is a function of the solvent viscosity. However, in a higher dimension, the dynamic diffusion coefficient depends only on the membrane bare friction.

IV. NONZERO CONCENTRATION

In Sec. IV A the dynamic scaling theory for a single membrane dissolved in solution is extended to nonzero concentration. The *m*-membrane free energy is proposed and the renormalized draining parameter and excluded volume strength are introduced. In Θ solvents, only one renormalized draining parameter is taken to describe the transition from nondraining to free-draining behavior. However, for good solvents, based on the effective medium argument, both the renormalized draining parameter and the excluded volume strength are required. The arguments presented below deal mainly with the behaviors of the specific viscosity and diffusion coefficient.

For a membrane solution with a nonzero concentration, the membrane-membrane interaction starts to set in. We generalize the one membrane free energy [Eq. (1)] to the *m*-membrane free energy, and the intramembrane and intermembrane excluded volume effects are included. Hence the dynamic scaling relationship of the nonzero concentration membrane solution follows an extension of the preceding work on the single membrane free energy. We first write down the *m*-membrane free energy as [17],

$$\frac{F_m}{k_BT}=\frac{1}{2}\kappa\sum_{\alpha=1}^{m}\int_0^{L^D}d^Dx\sum_{a=1}^{D}\left(\frac{\partial\vec{R}_\alpha}{\partial x_a}\right)^2+\frac{1}{n!}v_n\sum_{\substack{\alpha,\beta=1\\ \alpha\neq\beta}}^{m}\int_0^{L^D}\prod_{i=1}^{n}d^Dx_i\prod_{j=1}^{n-1}\delta^d(\vec{R}_\alpha(x_j)-\vec{R}_\beta(x_n')). \tag{34}$$

Following the same procedure in Sec. II, we then perform the nonzero membrane concentration Kirkwood diffusion equation

$$\left\{\frac{\partial}{\partial t}-\sum_{\alpha,\beta=1}^{m}\int_0^{L^D}d^Dx\int_0^{L^D}d^Dx'\frac{\delta}{\delta\vec{R}_\beta}\times\left[\frac{k_BT}{\zeta_0}\mathbb{1}\delta^D(x-x')\,\delta_{\alpha\beta}+\frac{\rho_0^2k_BT}{\eta_0}T(\vec{R}_\alpha(x)-\vec{R}_\beta(x'))\right]\times\left[\frac{\delta}{\delta\vec{R}_\alpha}+\frac{\delta\overline{F}}{\delta\vec{R}_\alpha}\right]\right\}P=\delta^D(x-x'), \tag{35}$$

where $\overline{F}_m=F_m/k_BT$. Moreover, applying the scale transformations in Sec. II to the free energy in Eq. (34) leads to the homogeneity relation

$$F_m(\vec{R}_\alpha,L,l,v_n)=F_m(\vec{R}_\alpha',Nb,1,v_n\times l^{(n-2)D-(n-1)d}b^{(n-1)(2-D)d/2-nD}), \tag{36}$$

and the time scales obey the scaling relationships

$$
\begin{aligned}
&t\Phi\left(\frac{\delta}{\delta\vec{R}_\alpha},\vec{R}_\alpha,L,l,v_n,\frac{k_BT}{\zeta_0},\frac{\rho_0^2k_BT}{\eta_0}\right)\\
&=t''\Phi\left(\frac{\delta}{\delta\vec{R}'_\alpha},\vec{R}'_\alpha,Nb,1,v_nl^{(n-2)D-(n-1)d}b^{(n-1)(2-D)d/2-nD},1,\frac{\zeta_0}{\eta_0}\frac{l^{D+2-d}}{b^{D+(2-D)(2-d)/2}}=h_<\right)\quad \text{for}\quad d<\frac{4}{2-D}\\
&=t'\Phi\left(\frac{\delta}{\delta\vec{R}'_\alpha},\vec{R}'_\alpha,Nb,1,v_nl^{(n-2)D-(n-1)d}b^{(n-1)(2-D)d/2-nD},1,\frac{\eta_0}{\zeta_0}l^{d-2-D}b^{D-(2-D)(d-2)/2}=h_>\right)\quad \text{for}\quad d>\frac{4}{2-D}.
\end{aligned}
\tag{37}
$$

Here Φ represents the evolution equation of the second term on the left hand side of Eq. (35). Therefore, Eqs. (36) and (37) provide us with the scaling variables for the dynamic scaling functions.

With the above argument of the homogeneity relation, we are ready to discuss the dynamic properties. In the finite concentration membrane solution, due to the membrane-solution and membrane-membrane interactions, hydrodynamic interaction sets in and then the draining parameter is required. This draining parameter shows quite a different behavior for $c=0$ and $c\neq 0$. For example, in the dimension $d>4/(2-D)$, we use $h=Y$ for $c=0$ and $h_\rangle=Y^{-1}$ for $c\neq 0$. Conversely, in the dimension $d<4/(2-D)$, we choose $h=Y^{-1}$ for $c=0$ and $h_\langle=Y$ for $c\neq 0$. Recall that, in the work of MD, the time scales separated by $d=4$ are not valid when the polymer scaling relationship is extended to a membrane solution with non-zero concentration.

A. Specific viscosity

In this section, we study the dynamic scaling behavior of the specific viscosity for Θ solvents and good solvents in a finite concentration membrane solution. Effective medium arguments are invoked for the solvent influences. Hence the draining parameter and excluded volume effect are screened by membrane-membrane interactions and membrane-solvent interactions.

The specific viscosity $\eta_{\rm sp}$ can now be generalized, due to the linear response theory, to

$$
\eta_{\rm sp}(c)=\frac{1}{k_BT\eta_0V}\int_0^\infty dt\langle J(t)\cdot J(0)\rangle,
\tag{38}
$$

where V is the system volume and the momentum flux tensor is defined by $J(t)=-k_BT(l/b)^D\int_0^{(Nb)^D}d^Ds'\vec{R}'_{\alpha x}(s',t')\,\delta/(\delta\vec{R}'_{\alpha y})\overline{F}_m(\vec{R}'_B,L',l',v'_n)$. In this scenario, the scaling forms for the specific viscosity become

$$
\begin{aligned}
\eta_{\rm sp}(c)&=\frac{\zeta_0}{\eta_0V}l^{2+d}b^{-2}\hat{f}_{\eta_{\rm sp}}\left(m,Nb,v_nl^{(n-2)D-(n-1)d}b^{(n-1)(2-D)d/2-nD},\frac{\zeta_0}{\eta_0}l^{D+2-d}b^{-D-(2-D)(2-d)/2},Vb^{d/2}l^{-d}\right),\\
&\qquad \text{for}\quad d<\frac{4}{2-D}\\
&=\frac{l^d}{Vb^{(2-D)d/2}}\hat{f}_{\eta_{\rm sp}}\left(m,Nb,v_nl^{(n-2)D-(n-1)d}b^{(n-1)(2-D)d/2-nD},\frac{\eta_0}{\zeta_0}l^{d-2-D}b^{D-(2-D)(d-2)/2},Vb^{d/2}l^{-d}\right),\\
&\qquad \text{for}\quad d>\frac{4}{2-D}.
\end{aligned}
\tag{39}
$$

We can now introduce the membrane concentration via the volume relation $(m/V)(b^{1/2}l^{-1})^d=c(N_A/M)(b^{1/2}l^{-1})^d$ and set $Nb=1$. Then Eq. (39) is reduced to

$$
\begin{aligned}
\frac{n_{\rm sp}(c)}{c}&=\frac{\zeta_0}{\eta_0}\frac{N_A}{M_A}l^{2+D}N^2\hat{f}_{\eta_{\rm sp}}\left(\frac{c}{c_0^*},h_\langle,Z\right)\quad \text{for}\quad d<\frac{4}{2-D}\\
&=\frac{N_A}{M_A}l^dN^{(2-D)d/2}\hat{f}_{\eta_{\rm sp}}\left(\frac{c}{c_0^*},h_\rangle,Z\right)\quad \text{for}\quad d>\frac{4}{2-D},
\end{aligned}
\tag{40}
$$

where the critical concentration c_0^* is denoted as $(b^{1/2}l^{-1})^dM_A/N_A$. This characteristic concentration c_0^* is recognized as the membrane beginning to interpenetrate in a Θ solvent, and the draining parameter which measures where the hydrodynamic interaction starts to appear. In the present case, we require that the power of N in the draining parameter is positive when $c \neq 0$. However, as the solution concentration is decreased to zero, the draining parameter shows an inverse behavior, i.e., the draining parameter approaches zero as N increased. This means that the powers of N in $h_{\substack{<\\>}}$ are negative.

Θ *solvents*

For a Θ solvent, we mean that both the intermembrane and intramembrane bead-bead interactions vanish. Typically, only hydrodynamic interactions are screened in this solution. One of the synthesized tethered membrane, for example, is the cross-linked poly(methyl methacrylate) [19,29]. Since acetone and toluene, etc, have been used as the Θ solvent for poly(methyl methacrylate) [30], and they could be a good choice as Θ solvents for the cross-linked poly(methyl methacrylate). Setting $v_n = 0$, the scaling relation for the specific viscosity is simplified to

$$\frac{\eta_{\mathrm{sp}}(c)}{c} = \frac{\zeta_0}{\eta_0}\frac{N_A}{M_A} l^{2+D} N^2 \hat{f}_{\eta_{\mathrm{sp}}}\left(\frac{c}{c_0^*}, h_{\langle}, 0\right), \quad \text{for } d < \frac{4}{2-D}. \tag{41}$$

At infinite dilution $c \rightarrow 0$, the only relevant variable is the draining parameter $h_{\langle}$. Moreover, in the limit $N \rightarrow \infty$ or $h_{\langle} \rightarrow \infty$, the self-friction ζ_0 becomes irrelevant. This ensures that $\hat{f}_{\eta_{\mathrm{sp}}}$ obeys the scaling from $\hat{f}_{\eta_{\mathrm{sp}}}(0,q,0) \sim q^{w_4}$ while w_4 is determined by the requirement that $\eta_{\mathrm{sp}}(0) = c[\eta]$ is independent of ζ_0. The appropriate choice of w_4 is -1. Hence we have

$$[\eta] = a_{\eta} \frac{N_A}{M_A} l^d N^{2-d(2-D)/2} = \frac{a_n}{c_0^*}, \tag{42}$$

where $a_{\eta} = \text{const}$ and $c_0^* = c_0^*/[\eta]$. This is the Rouse-Zimm result. Note that c_0^* and $[\eta]$ are interchangeable.

As the membrane concentration increases, the hydrodynamic screening effect starts to set in. There are two renormalized screening parameters such as the draining strength and the excluded volume effect. We choose $h_{\langle}(c)$ as the screened hydrodynamic interaction, i.e., $\eta_{\mathrm{sp}}(c)/c = (\zeta_0/\eta_0) \times (N_A/M_A) l^{2+D} N^2 \hat{f}_{\eta_{\mathrm{sp}}}(h_{\langle}(c))$. Comparing to Eq. (41) leads to the general expression $\eta_{\mathrm{sp}}(c)/c = (\zeta_0/\eta_0) \times (N_A/M_A) l^{2+D} N^2 \hat{f}_{\eta_{\mathrm{sp}}}(h_{\langle} g_{\eta}(c[\eta]))$. This defines the draining strength, $h_{\langle}(c) = h_{\langle} g_{\eta}(c[\eta])$, which describes the strength of the hydrodynamic interaction. However, the boundary condition of $g_{\eta}(c[\eta])$ remains to be determined. At $c = 0$, we require that $h_{\langle}(0)$ is equal to $h_{\langle}$, i.e., $g_{\eta}(0) = 1$. Conversely, as $c \rightarrow \infty, \eta_{\mathrm{sp}}$ reduces to the Rouse result, i.e., $\eta_{\mathrm{sp}}(\infty) \sim c(\zeta_0/\eta_0)(N_A/M_A) l^{2+D} N^2$. Thus we have $g_{\eta}(\infty) = 0$, i.e., $\hat{f}_{\eta_{\mathrm{sp}}}(h_{\langle} g_{\eta}(\infty)) \sim 1$.

With the preceding boundary conditions, we are more interesting in the general form of g_{η}. By using the argument of an effective medium, $h_{\langle}(c)$ is treated as the draining strength for a single membrane dissolved in the effective medium generated by the solvent and all the other membranes. Hence, by adding a small amount of membrane δc to the solution, the draining parameter is changed from $h_{\langle}(c)$ to $h_{\langle}(c) g_{\eta}(\delta c[\eta])$. Then the concentration dependent draining parameter obeys the statistical relation, i.e., $h_{\langle}(\overline{c}) = h_{\langle}(c + \delta c) = h_{\langle}(c) g_{\eta}(\delta c[\eta])$ or $g_{\eta}((c+\delta c)[\eta]) = g_{\eta}(c[\eta]) g_n(\delta c[\eta])$. The general form of the solution of g_{η} can be expressed as $g_{\eta}(c[\eta]) = e^{-\alpha c[\eta]}$ where α is a positive constant. We then have the specific viscosity $\eta_{\mathrm{sp}}(c)/c = (\zeta_0/\eta_0)(N_A/M_A) l^{2+D} N^2 \hat{f}_{\eta_{\mathrm{sp}}}(h_{\langle} e^{-\alpha c[\eta]})$. For a sufficiently large membrane and at very low concentration, the condition $h_{\langle} e^{\alpha c[\eta]} \gg 1$ suggests a power law form for the scaling function $\hat{f}_{\eta_{\mathrm{sp}}}(h_{\langle} e^{-\alpha c[\eta]}) \sim (h_{\langle} e^{-\alpha c[\eta]})^{w_5}$, where w_5 is determined by the requirement that in the asymptotic regime $\eta_{\mathrm{sp}}(c)/c|_{c \rightarrow 0} = c[\eta]$ is independent of ζ_0. Therefore, we find that $w_5 = -1$, and the specific viscosity becomes

$$\eta_{\mathrm{sp}}(c) = c[\eta] e^{\alpha c[\eta]}. \tag{43}$$

This case shows a non-power-law form derived from the scaling function, and is valid only for a large membrane. Expanding the exponential part in terms of the small concentration, we finally obtain the limiting law $\eta_{\mathrm{sp}}(c) = c[\eta](1 + \alpha c[\eta] + O(c^2))$ for the specific viscosity. Experimentally, α is identified as Huggins coefficient.

Now we turn to consider the higher dimensional case, i.e., $d > 4/(2-D)$. By setting $v_n = 0$, the specific viscosity becomes

$$\frac{\eta_{\mathrm{sp}}(c)}{c} = \frac{N_A}{M_A} l^d N^{(2-D)d/2} \hat{f}_{\eta_{\mathrm{sp}}}\left(\frac{c}{c_0^*}, h_{\rangle}, 0\right). \tag{44}$$

For a nonzero concentration solution $c \rightarrow 0$ and large bead number $N \rightarrow \infty$, or $h_{\rangle} \rightarrow \infty$, the specific viscosity obeys the scaling form, i.e., $\eta_{\mathrm{sp}}(c)/c|_{c \rightarrow 0} = [\eta] = (N_A/M_A) l^d N^{(2-D)d/2} h_{\rangle}^{w_6}$. The limiting behavior is expected to be proportional to ς_0/η_0, and hence the power w_6 is equal to -1. Therefore, we find that the Rouse result $[\eta] = a_{\eta}/c_0^*$ is recovered.

Now we turn to consider the higher concentration membrane solution. Substituting $h_{\rangle}(c)$ into Eq. (40) leads to $\eta_{\mathrm{sp}}(c)/c = (N_A/M_A) l^d N^{d(2-D)/2} \hat{f}_{\eta_{\mathrm{sp}}}(h_{\rangle} g_{\eta}(c[\eta]))$ where the definition of $h_{\rangle}(c) = h_{\rangle} g_{\eta}(c[\eta])$ is employed. In particular, the boundary condition of $g_{\eta}(c[\eta])$ at $c = 0$ is $g_{\eta}(0) = 1$. As the membrane concentration approaching infinity, the entanglement effect can be ignored, and $\eta_{\mathrm{sp}}(\infty) \propto c(N_A/M_A) l^d N^{(2-D)d/2} \hat{f}_{\eta_{\mathrm{sp}}}(0)$, i.e., $g_{\eta}(\infty) = 0$. Hence the general solution has the unique form $h_{\rangle}(c) = h_{\rangle} e^{-\alpha' c[\eta]}$, where α' is a positive constant. The specific viscosity in the higher dimension then satisfies the following scaling form:

$$\frac{\eta_{\mathrm{sp}}(c)}{c} = \frac{N_A}{M_A} l^d N^{(2-D)d/2} \hat{f}_{\eta_{\mathrm{sp}}}(h_{>} e^{-\alpha' c[\eta]}). \tag{45}$$

For a sufficiently large membrane and dilute solution, the condition $h_{\rangle} e^{-\alpha' c[\eta]} \gg 1$ implies a power law form for the scaling function $\hat{f}_{\eta_{\mathrm{sp}}}(h_{\rangle}e^{-\alpha' c[\eta]}) \sim (h_{\rangle}e^{-\alpha' c[\eta]})^{w_7}$, whereas w_7 is determined by the requirement similar to Eq. (43). Consequently, we realize that the power w_7 is equal to -1, and the specific viscosity becomes $\eta_{\mathrm{sp}}(c) = c[\eta]e^{\alpha' c[\eta]}$. In the higher dimension, we also evaluate the non-power-law form for the scaling function. By expanding the exponential part in terms of a small concentration, a similar limiting behavior of $\eta_{\mathrm{sp}}(c)$ in $d < 4/(2-D)$ is obtained as $\eta_{\mathrm{sp}}(c) = c[\eta](1+\alpha' c[\eta]+O(c^2))$.

Good solvents

In good solvents, both the hydrodynamic interaction and the excluded volume effect are screened, i.e., $h_{\substack{<\\>}}$ and v_n are renormalized. In this context, the general expression of the specific viscosity in good solvents is then modified by the concentration, and is expressed as

$$\frac{\eta_{\mathrm{sp}}(c)}{c} = \frac{\zeta_0}{\eta_0}\frac{N_A}{M_A} l^{2+D} N^2 \hat{f}_{\eta_{\mathrm{sp}}}\left(h_{\langle} g_{\eta,1}\left(\frac{c}{c_0^*}, Z\right), Z g_{\eta,2}\left(\frac{c}{c_0^*}, Z\right)\right), \tag{46}$$

when $d < 4/(2-D)$. In the above equation, the intermembrane excluded volume effect is presented in $g_{\eta,1}$ and $g_{\eta,2}$. Similarly, the intramembrane excluded volume effect is displayed in Z. As usual, applying the power law form to each individual concentration correction terms $g_{\eta,i} = \overline{g}_{\eta,i}[(c/c_0^*)Z^{w_8}]$, where $i=1$ and 2, and under the conditions $Z \gg 1$ and $(1/c_0^*)Z^{w_8} \sim N^{3\nu}$, we obtain the power of the argument of $g_{\eta,i}$ as

$$w_8 = \frac{3\nu - d(D-2)/2}{nD-(n-1(2-D)d/2}.$$

In addition, in good solvents, the above specific viscosity becomes

$$\frac{\eta_{\mathrm{sp}}(c)}{c} = \frac{\zeta_0}{\eta_0}\frac{N_A}{M_A} l^{2+D} N^2 \hat{f}_{\eta_{\mathrm{sp}}}\left(h_{\langle} \overline{g}_{\eta,1}\left(\frac{c}{c^*}\right), Z\overline{g}_{\eta,2}\left(\frac{c}{c^*}\right)\right), \tag{47}$$

where c^* is the characteristic concentration[4] and is equal to

$$(M_A/N_A) l^d N^{-3\nu}$$
$$\times (v_n l^{(n-2\_D-(n-1)d})^{-[3\nu-d(D-2)/2]/[nD-(n-1)(2-D)d/2]}.$$

In particular, we adopt the zero-concentration boundary conditions as $\overline{g}_{\eta,1}(0) = \overline{g}_{\eta,2}(0) = 1$. In order to recover our preceding infinite dilution result, the limiting law of specific viscosity becomes $\eta_{\mathrm{sp}}(c)/c|_{c\to 0} = (\zeta_0/\eta_0) \times (N_A/M_A) l^{2+D} N^2 \hat{f}_{\eta_{\mathrm{sp}}}(h_{\langle}, Z)$. As $N \to \infty$ and at infinite dilution, $\hat{f}_{\eta_{\mathrm{sp}}}(h_{\langle}, Z)$ can be expressed in terms of $(1/h_{\langle})\hat{f}_{\eta_{\mathrm{sp}}}(Z)$, when $h_{\langle} \gg 1$. When the excluded volume effect is strong, the $\hat{f}'_{\eta_{\mathrm{sp}}}(Z)$ term approaches the power law form Z^{w_g} as $Z \gg 1$. The final result of the limiting law of specific viscosity, i.e.,

$$\left.\frac{\eta_{\mathrm{sp}}(c)}{c}\right|_{c\to 0} = [\eta] = a'_{\eta}/c^*, \tag{48}$$

is in good agreement with the result of polymer theory. Due to the properties of $g_{\eta,i}(c[\eta]) = \exp(-\alpha_i c[\eta])$ and $\overline{g}_{\eta,i}(c/c^*) = g_{\eta,i}(c[\eta])$ when $c \to \infty$, $\eta_{\mathrm{sp}}(\infty)$ should be proportional to $(\zeta_0 N_A/\eta_0 M_A) l^{2+D} N^2$. Hence the α_i are positive constants. Obviously, the limiting law of specific viscosity is then reduced to $\eta_{\mathrm{sp}}(c)/c = (\zeta_0 N_A/\eta_0 M_A) l^{2+D} N^2 \hat{f}_{\eta_{\mathrm{sp}}}(h_{\langle}e^{-\alpha_1 c[\eta]}, Ze^{-\alpha_2 c[\eta]})$. If both $h_{\langle}e^{-\alpha_1 c[\eta]}$ and $Ze^{-\alpha_2 c[\eta]}$ are much larger than 1, then, in good solvents, we can ignore one of α_i. Thus we have the exponential relationship of the nonzero concentration specific viscosity

$$\frac{\eta_{\mathrm{sp}}(c)}{c} = [\eta]\exp(\alpha_1 c[\eta] - w_9 \alpha_2 c[\eta]), \tag{49}$$

where $w_9 = [d\nu - (2-D)d/2]/[nD-(n-1)(2-D)d/2]$. The Huggins coefficient $\alpha_1 - w_9\alpha_2$ involves the hydrodynamic interaction and the excluded volume effect. Nevertheless, the related coefficients α_i cannot be solved by scaling theory. The limiting cases in Eqs. (48) and (42) are valid for a membrane solution at low concentration and large membranes.

Now we examine the specific viscosity when the external dimension is larger than $4/(2-D)$, i.e.,

$$\begin{aligned}\frac{\eta_{\mathrm{sp}}(c)}{c} &= \frac{N_A}{M_A} l^d N^{(2-D)d/2} \hat{f}_{\eta_{\mathrm{sp}}}\left(\frac{c}{c_0^*}, h_{\rangle}, Z\right)\\ &= \frac{N_A}{M_A} l^d N^{(2-D)d/2} \hat{f}_{\eta_{\mathrm{sp}}}\left(h_{\rangle} g_{\eta,1}\left(\frac{c}{c_0^*}, Z\right) Z g_{\eta,2}\left(\frac{c}{c_0^*}, Z\right)\right).\end{aligned} \tag{50}$$

In the present case, the demarcation between dilute and semidilute behaviors in good solvents occurs at a characteristic concentration, i.e., $c^* \sim M_A N^{-d\nu}$, which reflects the perturbed membrane dimension. Hence the power law form for $g_{\eta,i}$ follows from $g_{\eta,i} = \overline{g}_{\eta,i}[(c/c_0^*)Z^{w_{10}}]$ for $i=1$ and 2, and $Z \gg 1$, with the requirement of $c_0^* = (M_A/N_A) \times [l^d N^{d(2-D)/2-d}]^{-1}$ and $(1/c_0^*)Z^{w_{10}} \sim N^{-d(1-\nu)}$. We find, in particular, that w_{10} is equal to $[d(\nu-(2-D)/2)]/[nD-(n-1)(2-D)d/2]$. Note that the bead number in each dimension is N. The total number of beads of the tethered membrane is N^d. So the demarcation concentration is different from the polymer solution. The specific viscosity is then reduced to $\eta_{\mathrm{sp}}(c)/c = (N_A/M_A) l^d N^{(2-D)d/2} \hat{f}_{\eta_{\mathrm{sp}}}(h_{\rangle}\overline{g}_{\eta,1}(c/c^*), Z\overline{g}_{\eta,2}(c/c^*))$ which recovers the infinite dilution result $[\eta] = [\eta_{\mathrm{sp}}(c)/c]|_{c\to 0} = (N_A/M_A) l^d N^{d(2-D)/2} \hat{f}_{\eta_{\mathrm{sp}}}(h_{\rangle}, Z)$. In addition, we choose the boundary conditions $\overline{g}_{\eta,1}(0) = \overline{g}_{\eta,2}(0) = 1$. At infinite dilution, $[\eta]$ is proportional to ζ_0/η_0, as we obtained in Sec. III B, which results in $\hat{f}_{\eta_{\mathrm{sp}}}(h_{\rangle}, Z) = (1/h_{\rangle})\hat{f}'_{\eta_{\mathrm{sp}}}(Z)$ when $h_{\rangle} \gg 1$. When the excluded volume effect is strong, i.e., $Z \gg 1$ and $\hat{f}'_{\eta_{\mathrm{sp}}}(Z)$

$\propto Z^{(d\nu-2)/[nD-(n-1)(2-D)d/2]}$, it is verified that $[\eta]$ is not equal to a'_{η}/c^*. Note that the interchangeability scaling behavior between c^* and $[\eta]$ is no longer obeyed in higher dimension. Clearly, the $\overline{g}_{\eta,i}(c/c^*)$ are obtained by invoking the effective medium arguments leading to $\overline{g}_{\eta,i}(c/c^*) = g_{\eta,i}(c[\eta]) = e^{-\alpha_i' c[\eta]}$. The requirement, at very high membrane concentration, of $\eta_{\mathrm{sp}}(\infty) \propto c(N_A/M_A) l^d N^{d(2-D)/2}$, indicates that α_i' are all positive. Consequently, we have the concentration dependent expression of the specific viscosity $\eta_{\mathrm{sp}}(c)/c = (N_A/M_A) l^d N^{d(2-D)/2} \hat{f}_{\eta_{\mathrm{sp}}}(h_{\rangle} e^{-\alpha_1' c[\eta]}, Z e^{-\alpha_2' c[\eta]})$. Moreover, when both $h_{\rangle} e^{-\alpha_1' c[\eta]}$ and $Z e^{-\alpha_2' c[\eta]}$ are larger than 1 we can simplify the specific viscosity into $\eta_{\mathrm{sp}}(c) = c[\eta] e^{k_H' c[\eta]}$ which suggests that

$$k_H' = \alpha_1' - \frac{d\nu - 2}{nD-(n-1)(2-D)d/2}\,\alpha_2' .$$

Here k_H' involves the contribution from both hydrodynamic and excluded volume screening effects.

B. Diffusion coefficient

In this section, we consider the diffusion coefficient for a tagged membrane in a finite concentration membrane solution. According to linear response theory, we now introduce the diffusion coefficient for a tagged membrane as

$$D_d(c) = \frac{1}{d}\int_0^\infty dt \langle V_\alpha(t) \cdot V_\alpha(0) \rangle, \tag{51}$$

where $V_\alpha(t)$ is the velocity of the center of mass of the tagged membrane, $(1/L^D)\int_0^{L^D} d^D x (\partial/\partial t)\vec{R}_\alpha(x,t)$. When the external dimension d is smaller than $4/(2-D)$, we employ the time scale $t'' = t(k_B T/\zeta_0)[b^2/(l^{2+D})]$ and obtain the concentration dependent diffusion coefficient $D_d(c) = (k_B T/\zeta_0) \times (N^{-D}/l^D) f_D(h_{\langle}, Z, c/c_0^*)$.

In Θ solvents, the draining parameter is defined by $h_{\langle}$ and the reduced concentration c/c_0^* is proportional to $c[\eta]$. The diffusion coefficient is then simplified to $D_d(c) = (k_B T/\zeta_0) \times (N^{-D}/l^D) f_D(h_{\langle}, 0, c/c_0^*)$. As usual, following the method of solving $\eta_{\mathrm{sp}}(c)$ in Θ solvents provides us the diffusion coefficient $D_d(c) = (k_B T/\zeta_0)(N^{-D}/l^D) f_D(h_{\langle} e^{-\overline{\alpha} c[\eta]})$ where $\overline{\alpha}$ is a positive constant. Hence, in the limiting case of a high concentration membrane solution, the Rouse behavior $D_d(\infty) = (k_B T/\zeta_0)(N^{-D}/l^D)$ is recovered. Moreover, the requirement that $D_s(c)$ is independent of ζ_0 for the condition $h_{\langle} e^{-\overline{\alpha} c[\eta]} \gg 1$ results in the asymptotic behavior of the dynamic diffusion coefficient as

$$D_d(c) = a_D \frac{k_B T}{\eta_0} l^{2-d} N^{(2-D)(2-d)/2} e^{-\overline{\alpha} c[\eta]}. \tag{52}$$

Conversely, at zero membrane concentration, the Rouse-Zimm result

$$D_d(0) = a_D \frac{k_B T}{\eta_0} l^{2-d} N^{(2-D)(2-d)/2} \tag{53}$$

is obtained.

The other interesting situation is the membrane dissolved in a good solvent. In this scenario, we may re-express the diffusion coefficient as $D_d(c) = (k_B T/\zeta_0) \times (N^{-D}/l^D) f_D(h_{\langle} e^{-\overline{\alpha} c[\eta]}, Z e^{-\overline{\alpha} c[\eta]}) = (k_B T/\zeta_0) \times (N^{-D}/l^D) f_D(q,z)$, where $\overline{\alpha_1}$ and $\overline{\alpha_2}$ are positive constants. In the limit $c \to 0$, the scaling function is independent of ζ_0, and the asymptotic form of $f_D(q,z)$ is expressed as $q f_D'(z)$ for $q \gg 1$ and the power law form for $f_D'(z)$ is z^w where $z \gg 1$. Since D_d is independent of ζ_0 and is proportional to $N^{-(d-2)\nu}$, we obtain the diffusion coefficient at infinite dilution as

$$\begin{aligned} D_d(c) = {} & a_D' \frac{k_B T}{\eta_0} l^{2-d} N^{(2-d)\nu} (v_n l^{(n-2)D-(n-1)d})^{[(2-d)(\nu-(2-D)/2)]/[nD-(n-1)(2-D)d/2]} \\ & \times \exp\left[-c[\eta] \left(\overline{\alpha_1} + \frac{(2-d)(\nu-(2-D)/2)}{nD-(n-1)(2-D)d/2} \overline{\alpha_2} \right) \right], \end{aligned} \tag{54}$$

where $a_D' = \mathrm{const}$, $h_{\langle} e^{-\overline{\alpha_1} c[\eta]} \gg 1$ and $Z e^{-\overline{\alpha_2} c[\eta]} \gg 1$. Obviously, expanding the above equation, we get the second virial correction (Higgins coefficient) $\overline{k_H} = \overline{\alpha_1} + \{[(2-d)(\nu - (2-D)/2)]/[nD-(n-1)(2-D)d/2]\} \overline{\alpha_2}$. The diffusion coefficient is then modified to

$$D_d(c)|_{\mathrm{exp}} = D_d(0)(1 - \overline{k_H} c[\eta] + O(c^2)). \tag{55}$$

The above equation is valid for low concentration or $h_{\langle} e^{-\overline{\alpha_i} c[\eta]} \ll 1$.

Now we consider a higher concentration of membrane solution. In this situation, both the hydrodynamic interaction and excluded volume effect should be screened. The validity conditions for membrane solution at high concentration are $e^{-\overline{\alpha_i'} c[\eta]} \ll 1$ where $i=1$ and 2; $\overline{\alpha_i'}$ are positive constants. Hence we are able to employ the renormalized parameters $h_{\langle}(c) = h_{\langle} g_{D,1}(c[\eta])$ and $v_n(c) l^{(n-2)D-(n-1)d} N^{nD-(n-1)(2-D)d/2} = Z g_{D,2}(c[\eta])$. The effective excluded volume strength and draining parameter no longer depend on N. Therefore, we require the power law form for $g_{D,i}(c[\eta])=a_{g_{D,i}}(c[\eta])^{w_i}$ where $a_{g_{D,i}}=\text{const}$, and the w_i are chosen in such a way that the effective number of the monomers experiencing a strong excluded volume effect and hydrodynamic interaction is independent of N. In other words, $h_{\langle}(c)$ obeys a power law form such as

$$h_{<}a_{g_1}\left[\frac{N_A}{M_A}l^d N^{d(2-D)/2-1}Z^{[d\nu-d(2-D)/2]/[nD-(n-1)(2-D)d/2]}\right]^{w_{11}}.$$

Since $c[\eta]$ is proportional to c/c_0^*, $h_{\langle}(c)$ is independent of N. Consequently, the power w_{11} is equal to $\frac{1}{2}([4-d(2-d)]/(d-d\nu))$ and the screened draining parameter $h_{\langle}(c)$ is denoted as

$$a_{g_{D,1}}\left(\frac{\zeta_0}{\eta_0}\right)l^{2+D-d}\left(\frac{cN_A}{M_A}l^d\right)^{(1/2)[4-d(2-d)]/(d-d\nu)}(\upsilon_n l^{(n-2)D-(n-1)d})^{(1/2)[d\nu-d(2-D)/2]/[nD-(n-1)(2-D)d/2][4-d(2-d)]/[d-d\nu]}.$$

On the other hand, the effective excluded volume strength $\upsilon_n(c)l^{(n-2)D-(n-1)d}\times N^{nD-(n-1)(2-D)d/2}$ can be expressed in terms of the power law form

$$Za_{g_{D,2}}\left[\left(\frac{cN_A}{M_A}l^d N^{(d/2)(2-D)-1}\right)Z^{[d\nu-d(2-D)/2]/[nD-(n-1)(2-D)d/2]}\right]^{w_{12}}.$$

Here we require that the effective excluded volume strength is independent of N. Therefore, we have the power

$$w_{12}=\frac{nD-(n-1)(2-D)d/2}{d-d\nu}.$$

Substituting these results back to the special viscosity in Sec. III B leads us to the specific viscosity

$$\frac{\eta_{\text{sp}}(c)}{c}=\frac{1}{a_{gD,1}}a_{gD,2}^{[d\nu-(2-D)d/2]/[nD-(n-1)(2-D)d/2]}\frac{N_A}{M_A}l^d N^2\left(\frac{cN_A}{M_A}l^d\right)^{-(d\nu-2)/(d\nu-d)}$$
$$\times(\upsilon_n l^{(n-2)D-(n-1)d})^{-[(d\nu-2)/(d\nu-d)][d\nu-d(2-D)/2]/[nD-(n-1)(2-D)d/2]}. \tag{56}$$

Furthermore, the power law form for the diffusion coefficient $D_d(c)$, in a good solvent, is simplified to $(k_BT/\zeta_0\kappa)l^{-D}N^{-D}qz^w$, where $q=h_{<}(c)$ and $z=\upsilon_n(c)$. With the requirement of $D_d(c)$ being independent of ζ_0 in good solvents, we have the final result of

$$D_d(c)=a_{gD,1}a_{gD,2}^{[(2-d)(\nu-(2-D)/2]/[nD-(n-1)(2-D)d/2]}\frac{k_BT}{\eta_0}l^{2-d}N^{-D}\left(\frac{cN_A}{M_A}l^d\right)^{-[D+(2-d)\nu]/(d\nu-d)}$$
$$\times(\upsilon_n l^{(n-2)D-(n-1)d})^{-[D+(2-d)\upsilon]/(d\nu-d)\times[d\nu-d(2-D)/2]/[nD-(n-1)(2-D)d/2]}. \tag{57}$$

Now we consider the diffusion coefficient for dimension d larger than $4/(2-D)$, and adopt the time scale $t'=t(k_BT/\eta_0)(b^{(2-D)/2}/l)^d$ in the linear response theory:

$$D_d(c)=\frac{k_BT}{\eta_0}l^{2-d}N^{(d/2-1)(D-2)}f\left(h_{\rangle},Z,\frac{c}{c_0^*}\right). \tag{58}$$

In the above equation, in Θ solvents, the draining parameter $h_{\rangle}$ and the reduced concentration $c/c_0^*\propto c[\eta]$ in Eq. (58) are expected. We therefore find that the diffusion coefficient can be reduced to $D_d(c)=(k_BT/\eta_0)l^{2-d}N^{(d/2-1)(D-2)}f_D(h_{\rangle}e^{-\overline{\alpha}'c[\eta]})$ which displays the Rouse behavior

$$D_d(\infty)\propto\frac{k_BT}{\eta_0}l^{2-d}N^{(d/2-1)(D-2)}, \tag{59}$$

when entanglement effects are ignored. Here $\overline{\alpha}'$ is a positive constant. The requirement of $D_d(c)$ being proportional to ζ_0^{-1} produces the asymptotic behavior $D_d(c)=(k_BT/\zeta_0)l^{-D}N^{-D}e^{-\overline{\alpha}'c[\eta]}$. Consequently, at $c=0$, the freely draining Rouse type result

$$D_d(0)=\frac{k_BT}{\zeta_0}l^{-D}N^{-D} \tag{60}$$

is recovered.

On the other hand, in good solvents, we modify both the hydrodynamic screening and excluded volume strengths by the concentration, and re-express them in terms of a power law form. Then the diffusion coefficient $D_d(c)$ is denoted as $(k_BT/\eta_0)l^{2-d}N^{(d/2-1)(D-2)}f_D(h_{\rangle}e^{-\overline{\alpha}_1'c[\eta]},Ze^{-\overline{\alpha}_2'c[\eta]})$ where

$\overline{\alpha_1'}$ and $\overline{\alpha_2'}$ are positive constants. As $c \to 0$, $D_d(c)$ is proportional to ζ_0^{-1}, the scaling function $f_D(q,z)$ can be expressed as $q f_D'(z)$ for $q \gg 1$, while

$$f_D'(z) \propto z^{[D-(d-2)\nu]/[nD-(n-1)(2-D)d/2]} \quad \text{for } z \gg 1.$$

Finally, the power law form for the diffusion coefficient becomes

$$D_d(c) = a_D' \frac{k_B T}{\zeta_0} l^{-D} N^{-(d-2)\nu} (v_n l^{(n-2)D-(n-1)d})^{[D-(d-2)\nu]/[nD-(n-1)(2-D)d/2]} \times \exp\left\{ -c[\eta]\left[\overline{\alpha_1'} - \frac{(d-2)\nu - D}{nD-(n-1)(2-D)d/2} \overline{\alpha_2'} \right] \right\} \tag{61}$$

where $a_D' = \text{const}$. The expansion of Eq. (61) in terms of the small concentration leads to the limiting law

$$D_d(c) = D_d(0)[1 - \overline{k_H'} c[\eta] + O(c^2)] \tag{62}$$

where the Huggins coefficient is

$$\overline{k_H'} = \overline{\alpha_1'} - \frac{(d-2)\nu - D}{nD-(n-1)(2-D)d/2} \overline{\alpha_2'}.$$

V. CONCLUSION

In conclusion, based on the free energy of a tethered membrane, we derive the membrane Langevin equation. The equivalent Kirkwood diffusion equations for the probability distributions are obtained for zero and finite concentration membrane solutions. Furthermore, there exist two time scales which are separated by the external dimension 4/(2 $-D$). The choice of the time scales depends on the convergence of the draining parameter. In this work, we solve the dynamic scaling behavior of the diffusion constant, specific viscosity, and structure factor in zero and nonzero concentration membrane solutions. In this respect, the dynamic scaling behavior of the tethered membrane depends on two fundamentally important effects, namely, the hydrodynamic interaction and the excluded volume strength. In Θ solvents, only the hydrodynamic interaction sets in. However, in good solvents, both the hydrodynamic interaction and the excluded volume effect need to be considered. These two effects are renormalized when the membrane concentration is increased. The screening effect is due to the coherence length of the associated hydrodynamic interaction being reduced, so that the net intramembrane and intermembrane excluded volume interactions have disappeared. Furthermore, our scaling theory is based on the effective medium argument. A short wavelength cutoff deserves a full renormalization group study. The more interesting behavior of a tethered membrane in a poor solvent is under investigation.

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